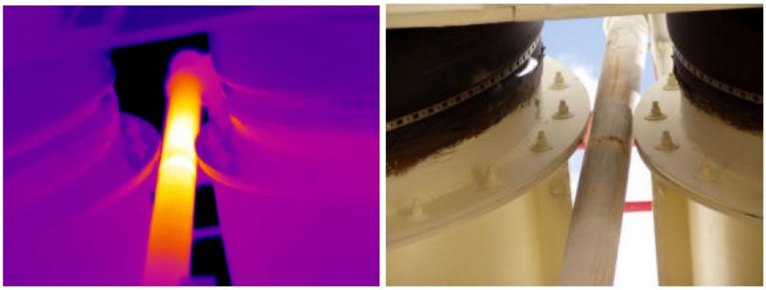
Temperature Measurement



Images provided courtesy of Brady Infrared

量測原理與機工實驗 (II)

熱流量測(2)

May, 23, 2012

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Why measuring temperature?

- one of the fundamental quantities:
  - Time
  - Length
  - Mass
  - Temperature:

    an *intrinsic* property of a system

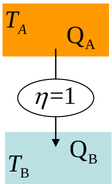
$$L_{A+B} = L_A + L_B$$

$$m_{A+B} = m_A + m_B$$

$$T_{A+B} \neq T_A + T_B$$

- Temperature scale: after Lord Kelvin (1984)
  - Carnot cycle at thermal equilibrium

$$Q_A / Q_B = T_A / T_B$$



  - Every temperature scale is with respect to a reference temperature, that is set to be the triple point of water at 1atm.

Temperature sensors

Employ propertiaes that change with temperature and can be easily quantified, including changes in (1) dimension, (2) electrical properties, (3) radiating characteristics, (4) physical states, (5) chemical states, …

- **Contact sensor**
  - -- Easy implementation
  - -- Direct measurement
  - -- Straightforward calibration

Operation limit, response time, …

- **Thermometer**
- **Bimetallic type**
- **Thermo-electric material**
- **Thermo-resistive material**
- **Thermocouple**
- **RTD**

Non-contact sensor

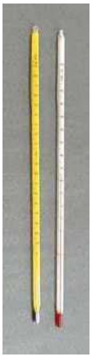
--Non-intrusive measurement
--Tolerance for high temperature level
-- Long distance monitoring
-- Better precision for some systems.

Difficult calibration, messy data extrapolation, maintenance, …

- **Pyrometer**
- **IR thermometer**
- **related imaging system**

Contact type (1) -- **thermal expansion method**

- Employ the knowledge of the thermal-mechanical property (mainly the thermal expansion) of solids, liquids or gases.
- The sensor performance highly depends on the material thermal properties.



1. Fluid-expansion thermometers (溫度計)

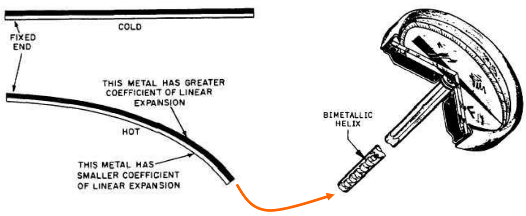
- – Mercury; alcohol; organic-liquid
- – Constant thermal expansion coefficient within the operating temperature range
- – Pros: do not require external power supply, stable, cheap, easy implementation
- – Cons: long response time, difficulty in remote monitoring, incapable of point measurement, limit by the boiling point of the filled liquid

* Error results from using different immersion from that used in calibration.

Contact type (1)-- **thermal expansion method**

2. Bimetallic sensor (雙金屬溫度計)

– Employ the difference in rate of thermal expansion between two metal strips, films,…etc



– due to its reversibility, frequently used as an <u>thermal on/off switch</u>

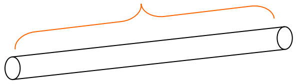
– low accuracy

Contact sensor (2): --Thermocouples TCs

• Use thermo-electric characteristics of a electrical conducting material, usually metallic wires

<u>**Seebeck effects**</u> : Non-zero electromotive force (emf) $\Delta E_\sigma(T)$ develops when the material experiences a temperature gradient along their length above absolute zero K:

$$\frac{\partial T}{\partial x} \neq 0$$



$$\Delta E_\sigma(T)$$

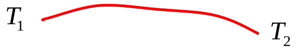
The magnitude of ΔE_σ depends on both

-- the material property, the <u>Seebeck coefficient</u> $\sigma(x,T) \equiv \frac{dE_\sigma}{dT}$

-- ambient temperature distribution $T(x)$

Contact sensor (2): **--Thermocouples TCs**

- Operation principle:

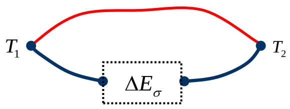


$$\sigma(x,T) \equiv \frac{dE_\sigma}{dT}$$

$$E_\sigma = \int_{T_0}^{T_2} \sigma(T)dT - \int_{T_0}^{T_1} \sigma(T)dT$$
$$= \int_{T_1}^{T_2} \sigma(T)dT$$
$$= E_\sigma(T_2) - E_\sigma(T_1)$$

Reference temperature
* Ice-bath junction
* Isothermal block

- Relative Seebeck coefficients σ_{AB}



$$E_{\sigma A} = \int_{T_1}^{T_2} \sigma_A(T)dT$$

$$E_{\sigma B} = \int_{T_1}^{T_2} \sigma_B(T)dT$$

$$\Delta E_\sigma = E_{\sigma A} - E_{\sigma B} = \int_{T_1}^{T_2} (\sigma_A - \sigma_B)dT = \int_{T_1}^{T_2} \sigma_{AB}(T)dT$$

$$\Delta E_\sigma = fn_1(T_2 \text{-} T_1)$$

$$T_2 \text{-} T_1 = fn_2(\Delta E_\sigma)$$

Knowledge of one end temperature is required.

**** All relative to some reference temperature.**

Contact sensor (2): **--Thermocouples TCs**

Significance of calibration

In-homogeneity in the environment
–local excessive temperature (greatly change the emf readouts)

Pre-existed mechanical/thermal stress in the set
–Due to fabrication, contaminants,…etc

Thermal resistance of the junction points
–Binding method does matter

- No theoretical models for the pre-existed stress; varying in-homogeneity in working environments. Thus, direct/ in situ calibration is often employed to offset the bias.

Contact sensor (2): **--Thermocouples TCs**

- Some facts
  - Common material pairs:
    - Platinum / rhodium (銠)
    - Chromel(鉻合金) / alumel (鋁合金)
    - Copper / constantan (康銅: 銅鎳合金)
    - Iron / constantan
  - Common junction methods
    - Welding, solding, binding, pressing, …etc

- TCs is an electrical conductor. So be aware of any power supply.
- The selection, installation details, and the operating conditions play a big role in obtaining an accurate and reliable measurement!

Contact sensor (3): **Thermal resistance type**

- Resistance Temperature Detectors are made of metals / alloys whose electrical resistance changes with temperature in a known fashion.

- Two big categories:

  1. Metallic devices (conventional **RTD**)
    - Materials whose resistance <u>increases</u> almost linearly with temperature (small and positive)
    - Platinum (PRTs):

$$R(T) = R(\mathrm{T}_0)\left[1 + aT + bT^2\right] \qquad -200^\circ C < T < 650^\circ C$$

$$a = 3.91\times10^{-3}\ {}^\circ C \qquad b = -5.78\times10^{-7}\ {}^\circ C^{-2}$$

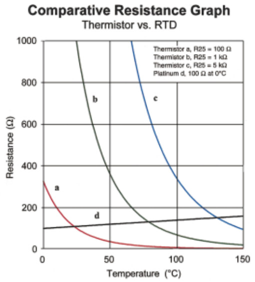
    - Cheaper materials:
      - Copper (Oxidation problem)
      - Nickel (strong non-linearity: narrow temperature ranges)
      - Silver, platinum

Contact sensor (3): **Thermal resistance type**

2. Ceramic **Semiconductor** devices (**thermistor**)

Resistance decreases non-linearly with temperature rise

$$R(T) = R(T_0)\, e^{\beta(1/T - 1/T_0)}$$



-- First generating a tiny current through the sensor.

-- Then use the resulting voltage drop to extrapolate temperature difference.

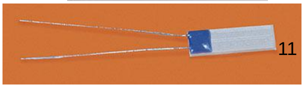
-- usually made of metallic oxides

Commercial types:

-- exposed to non-erosive fluid, gas

-- embedded in inert material with good thermal conductivity (pr

-- made of thin sheet for surface temperature measurement



Contact sensor (3): thermal resistance type

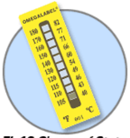
- In general, RTD gives accurate reading with <u>short response time (1/few secs)</u>
- As compared to TC,
  - Pros:
    - Stable output for a long time duration
    - Easy re-calibration
  - Cons:
    - Can't work on vibrating/deforming surfaces.
    - Require external power to generate the initial voltage drop.
    - Prone to errors due to
      - : Read-out wire (4-wire design to compensate the error)
      - : Insulation resistance after long-time operation (calibration)

Contact sensor (4): **Change-of-State device**

Made of labels, pellets, crayons, lacquers or liquid crystals whose appearance changes once a certain temperature is reached (phase-change temp).

Characteristics:

-- long response time (doesn't work for transient measurement)
-- low accuracy
-- irreversible process (except liquid-crystal displays)



TL-10 Change-of-State Temperature Measurement

Often apply when one-time permanent mark is desired.
For example, we want to make sure that the temperature of a piece of material has not exceeded a certain level during the product transportation.

Pyrometer—non-contact

- Related to the Greek words "pyros" and "metron" that mean "fire" and "to measure" respectively.
- Different types of pyrometers are designed to sample electromagnetic (EM) radiation in certain wavelengths to measure the energy magnitude or wavelength, which information is then used to determine the temperature.
  - Total radiation pyrometer: measure heating (due to total wave incidence) on the sensor (Thermopile)
  - Infrared pyrometer: similar to total radiation type, but restrict to infrared segment. Often use electronic detectors (Photocells)
  - Optical (Brightness) pyrometer: detect the changes in EM wavelength (in visible radiation range) due to temperature
- Recall that EM wave propagation doesn't require medium. Thus, pyrometer doesn't require contact with the source.

Basics of EM wave [1]

- All objects of temperature above absolute zero emit photon (energy, EM waves) according to its body temperature, shape, surface properties, ..etc.

Example: heating of a steel block:

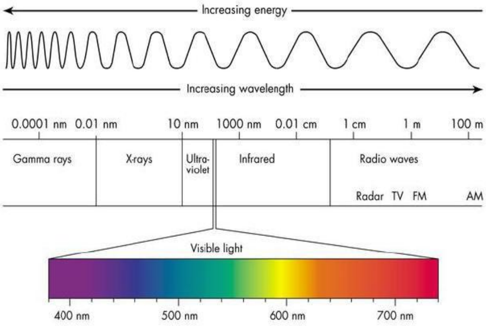
-- Below 550˚C, energy radiates in the form of 'heat'

We can 'feel' the existence of a massive hot object without touching it. Our skin works as the sensor.

-- Above 550˚C, block starts to glow (energy radiates in visible EM wave).

-- Keep increasing temperature, the color goes from a dull red, orange, yellow, and finally approaches white light at nearly melting temperature (about 1500˚C).

A sensor that responds to photon impact can then be used to detect temperature.



Basics of EM wave [2]

- In addition to energy emission, there is also energy absorption (α), reflection (ρ), and transmission (τ) at the surface. Energy conservation requires that α + ρ + τ = 1.
- A body must absorb energy before it can emit it. Thus, often assume that α is equal to ε (emission), giving, **ε + ρ + τ = 1**
  - Blackbody: τ → 1
  - Ideal radiator: ε → 1, **Stefan-Boltzmann law**:

Radiant heat transfer (W/m^2)

$$q = \sigma \varepsilon C_A (T_1^4 - T_2^4)$$

C_A, configuration factor, ε emission, σ Stefan-Boltzmann constant = 5.729 x10^(-8)

- **Plank's law:** For each temperature, there exists a particular wave length that the blackbody emits the maximum EM energy (peak intensity).

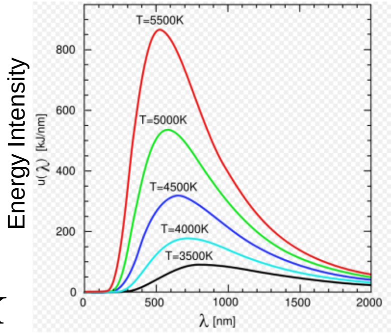
$$W_{B\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$

Basics of EM wave [3]

Wien's displacement law:

Increase of surface temperature induces shift of the wavelength of maximum emission towards the short-wave range:

$$\left(\lambda T\right)_{\max} \approx 2897.8\ \mu mK$$



- A photo-sensitive detector (Focal Plane Array—FPA) can be designed to locate the wavelength of maximum emission, which information is employed to back calculate the source temperature.

- Note that often it is the energy being measured. Energy only serves as a measure of surface temperature. It requires careful / extensive calibrations /correction to obtain the exact reading.

Basics of EM wave [4]

- Blackbody calibration
  - Blackened conical cavity that emits/absorbs radiations like a perfect blackbody (theoretical maximum).
  - Emissivity is defined as the ratio of true radiation to that from a blackbody:
    - Spectral (over a specific range of wavelength $\lambda_1 \rightarrow \lambda_2$)

$$\varepsilon_\lambda\left(\lambda,\mathrm{T}\right)=\frac{\mathrm{W_{true}}}{\mathrm{W_B}}=\frac{\int_{\lambda_1}^{\lambda_2}\mathrm{W}_{\mathrm{true}\lambda}(\lambda,\mathrm{T})\mathrm{d}\lambda}{\int_{\lambda_1}^{\lambda_2}\mathrm{W}_{\mathrm{B}\lambda}(\lambda,\mathrm{T})\mathrm{d}\lambda}$$

    - Total (over the entire spectrum)

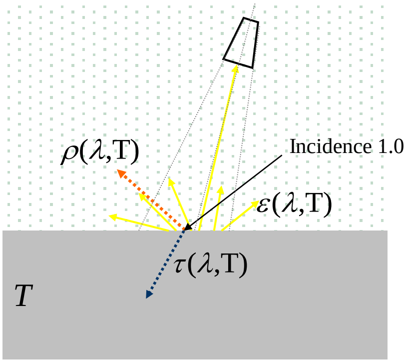
$$\varepsilon_{tot}\left(\lambda,\mathrm{T}\right)=\frac{\mathrm{W_{true}}}{\mathrm{W_B}}=\frac{\int_0^\infty\mathrm{W}_\lambda(\lambda,\mathrm{T})\mathrm{d}\lambda}{\int_0^\infty\mathrm{W}_{\mathrm{B}\lambda}(\lambda,\mathrm{T})\mathrm{d}\lambda}=\frac{\int_0^\infty\mathrm{W}_\lambda(\lambda,\mathrm{T})\mathrm{d}\lambda}{\sigma\mathrm{T}^4}$$

Tabulated values…, check your Heat Transfer textbook.

Radiation details…

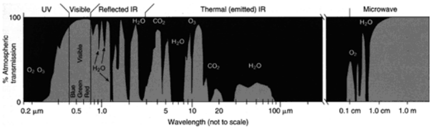
- Energy conservation: Sum of Emissivity, Reflectivity, Absorptivity

$$\varepsilon+\rho+\tau=1$$

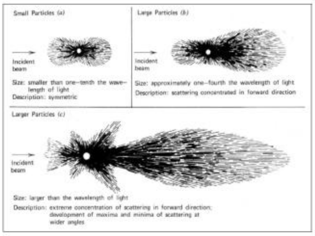


Before the IR is received by the detector, disturbances from the surroundings:

1. attenuated by the resonance absorption



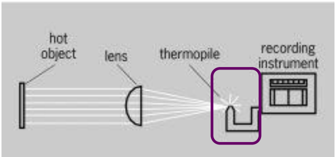
2. added noises due to the scattering effects of the droplets, carbon dioxide…



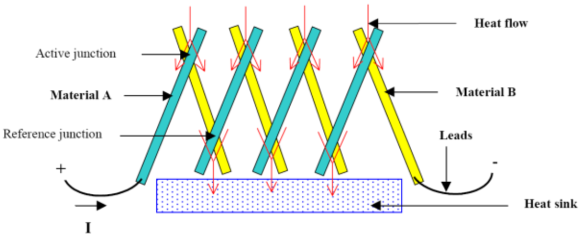
3. The filtering effects from the change in refractive index when IR passes the instrument lenses. Thus, specific operating ranges for commercial IR imaging systems. $0.7-20\mu m$[19]

Non-Contact sensor [1]: **Total-radiation pyrometer 全幅射高溫計**

- Key components: lens that collect the EM waves, an approximate blackbody receiver that helps to absorb all the incidence waves, a sensing unit (Thermopile 溫差電堆)



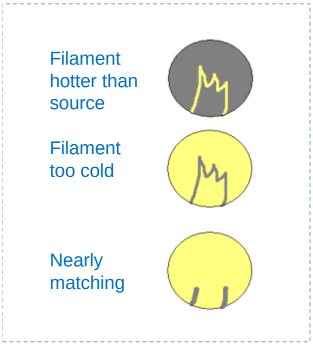
Receiver chamber

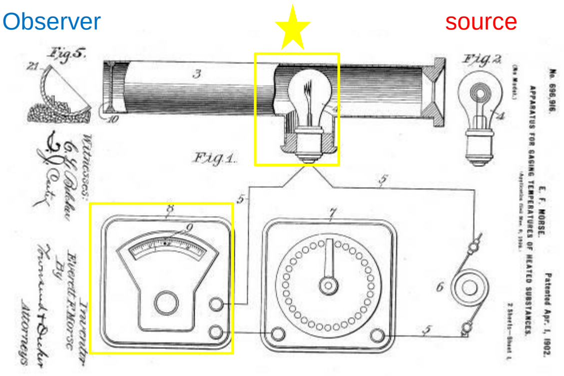


A thermopile is made of thermocouple junction pairs connected electrically in series. The absorption of thermal radiation by one of the thermocouple junctions, called the active junction, increases its temperature. The differential temperature between the active junction and a reference junction kept at a fixed temperature produces an electromotive force (emf) directly proportional to the differential temperature. This effect is called a thermoelectric effect.

Non-Contact sensor [2]: **Optical pyrometer**

Optical Pyrometers work on the basic principle of using the human eye to match the brightness of the hot object to the brightness of a calibrated lamp filament inside the instrument

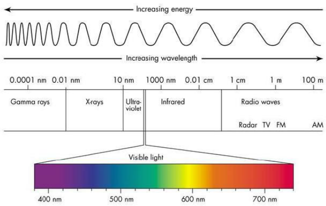




★ Lamp (filament) whose brightness can be tuned via the use of external power supply. The adjustment is calibrated with respect to a source with known temperature.

Non-Contact sensor [3]: **Infrared Thermograph**

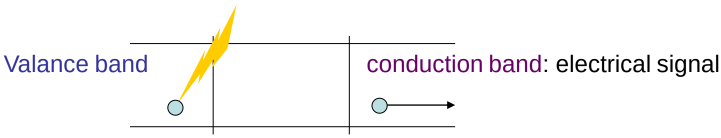
- Infrared (IR) is part of the electromagnetic spectrum (EM) whose wavelength falls in the range of $0.7-1000\mu m$



Photon detector (receiver):

-- Semi-conductor material

-- When the IR wave emitted from the surface provides sufficient energy for the electron of the detector to overcome the energy well, an electrical signal is generated.



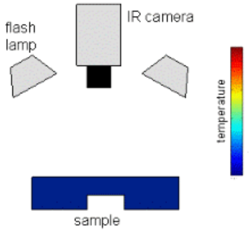
The signal is used to extrapolate the temperature profile of the emitting surface.

Non-Contact sensor [3]: **Thermography imaging system**

- A brief pulse of light is used to heat up the surface. As the sample cools, the internal fractures, voids or inclusions affects the heat flow and thus distorts the surface temperature distribution.

- An infrared camera (IR camera) records the surface temperature change.

- Combined with some thermographic signal reconstruction (TSR) method, fast, reliable and well-quantified data can be obtained for a Non-Destructive Testing (NDT) task.



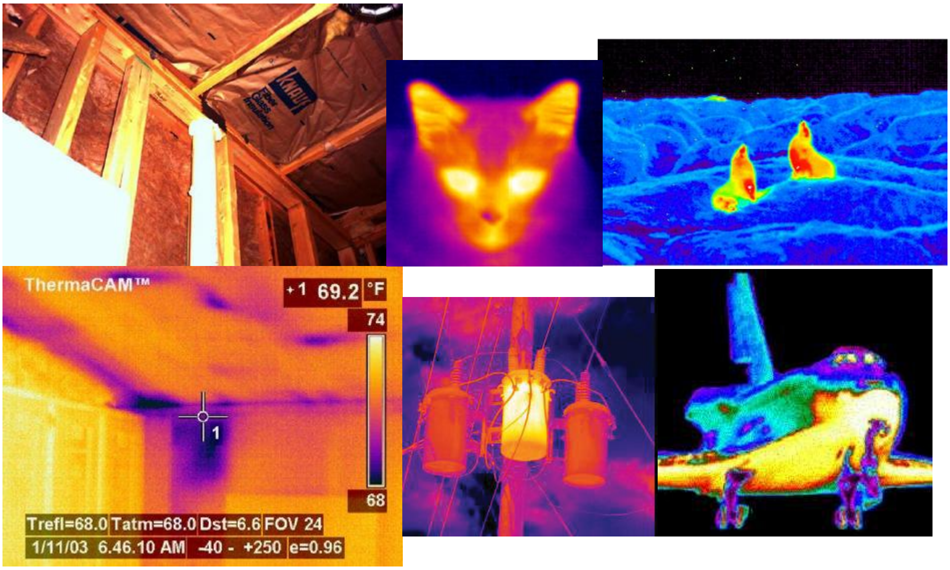
-- Pros:

Rapid response (molecular scale actions), high precision, large surface monitoring

-- Cons:

Expensive, difficult instrumentation, maintainance.

Non-contact sensor [3]: **IR Thermograph**

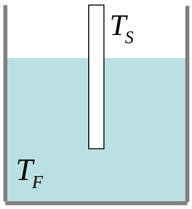


Error estimation

- Lack of calibration with respect to a known temperature reference.
- Instrumentation error
- Miscalculated working conditions:
  - IR wavelength window
  - energy well
  - thermal expansion coefficient…
- Wrongly estimated system characteristics:
  - Boundary conditions: insulated, constant influx, fixed temperature,…
  - Thermal properties: transport coefficients, contact resistances,
- Devices fatigue, chemical deposition and erosion,…

Transient measurement – **temperature**

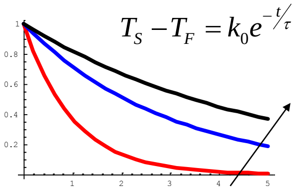
- Thermal expansion and a thermo-electric signal change results from energy transfer at molecular level. Thus the temperature raise is immediate.
- How fast a sensor can respond to a varying temperature field mainly depends on <u>the thermal characteristic of the “sensing assembly”</u>.



$$mc\frac{dT_S}{dt} = \dot{Q} = hS\left(T_F - T_S\right)$$

h : overall heat transfer coefficient
S : effective area

$$T_S - T_F = k_0 e^{-t/\tau}$$



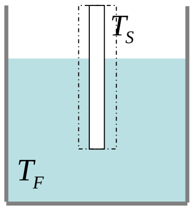
Increasing $\tau = \dfrac{mc}{hS}$

Response time

Transient measurement – **temperature**

- With outer layer

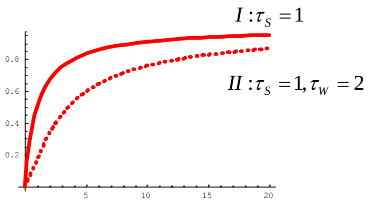
$$T_S - T_F = k_0 e^{-t/\tau} \qquad k_0 = \left(T_S - T_F\right)_{t=0}$$



$$T_S^* - T_F^* = e^{-t/\tau} = 1 - t/\tau + \left(t/\tau\right)^2 + \cdots$$

$$\left.\frac{T_S}{T_F}\right|_I \sim \frac{1}{\tau_S/t + \varepsilon}$$ -- small number, H.O.T.

$$\left.\frac{T_S}{T_F}\right|_{II} \sim \left(\frac{1}{\tau_S/t + \varepsilon}\right)\left(\frac{1}{\tau_W/t + \varepsilon}\right)$$



$$\tau_S = \frac{m_S c_S}{h_S S_S} \qquad \tau_W = \frac{m_W c_W}{h_W S_W}$$

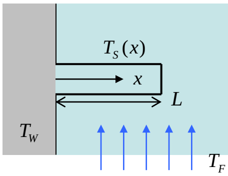
Transient measurement – **temperature**

- The thermal coupling between the layered materials usually are complicated. Thus, simplified theoretical model is in general insufficient (though characterizes the dynamic behavior).
- If accurate response is required, systematic experimental tests are employed to develop an empirical formulas.
- Furthermore, lab testing may not fully represent the thermal responses of the sensing assembly when they function in reality. Thus, in situ dynamic testing is again desired (in addition to calibration).
  - Especially important when the lab and actual facilities differ by several orders of magnitudes….

Theoretical analysis --error estimation/design criterion

- The previous analysis assumes a uniform temperature for the sensor.
- Heat conduction, convection, and radiation analysis can provide detailed temperature distribution along the element.

Forced convection over a probe

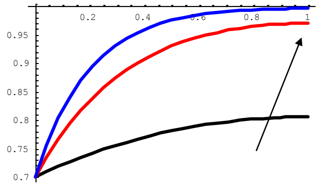


$$\frac{d^2 T_S}{dx^2} = \frac{hc}{kA}\left(T_S - T_f\right)$$

$$x = 0:\ T_S = T_W$$

$$x = L:\ \partial T_S / \partial x = 0$$

$$\frac{T_S(x)}{T_F} = 1 - \left(1 - \frac{T_W}{T_F}\right)\left[\frac{e^{-m(L-x)} + e^{m(L-x)}}{2\cosh mL}\right]$$



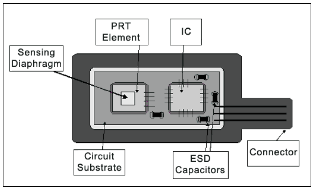
increasing $m = \sqrt{\frac{hc}{kA}}$

$$\frac{T_W}{T_F} = 0.7$$

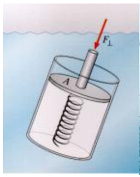
Pressure field measurement

Pressure-measuring transducer

- Pressure is often measured by transducing its effect to a deflection through use of a pressurized area and either a gravitational or elastic-restraining element.
- The deflection is often quantified by mechanical displacement, elastic strain, or piezoelectric response of the sensing element

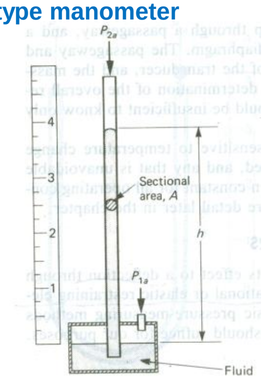


- Gravitational type:
  - Liquid column
  - Piston, dead-weights
- Direct-acting elastic type:
  - C-shaped Bourden tube
  - Elastic diaphragm
  - Bulk compression
  - Symmetrically or asymmetrically loaded tubes



Gravitational-type transducer

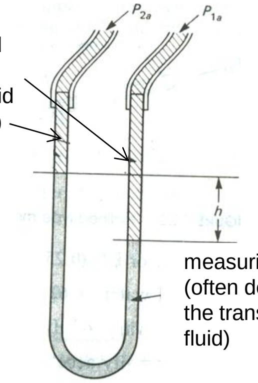
Well-type manometer



$$P_1 A = P_2 A + \rho g h A \Rightarrow P_1 - P_{atm} = \rho h g$$

Dual-fluid U-tube manometer

transmitting fluid (usually of the same type of fluid to be measured)



measuring fluid (often denser than the transmission fluid)

$$P_1 - P_2 = (\rho_m - \rho_t) h g$$

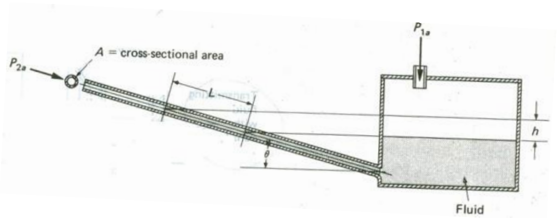
Cons: require consideration of temperature induced density variation;
breakage at high pressure environment

Gravitational-type transducer

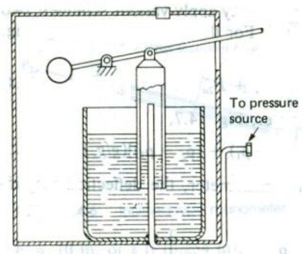
Often combined with <u>displacement amplification</u> for easier read-outs.

Dead-weight tester

$$P_{1a} - P_{2a} = \rho L \sin\theta g$$



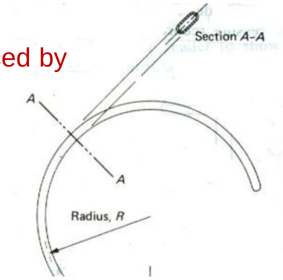
Inverted-bell system:
net weight of the bell depends on the depth of immersion



Good for static or quasi-static pressure measurement

Elastic-type transducer

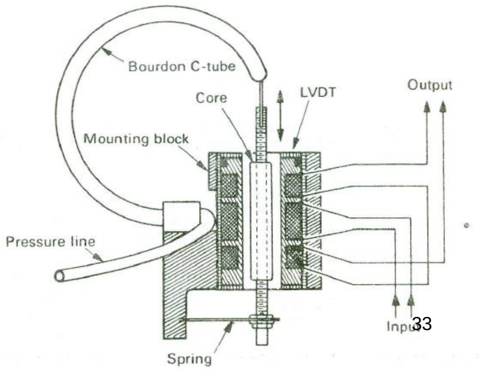
Force resulting from pressure application is balanced by elastic deflection or surface deformation.



[1] C-shaped Bourdon tube:

-- A coil with oval section tube when uncharged.

-- When pressure is applied to the tube, the oval section tends to round out. Since the inner and the outer arc lengths remain approximately equal to their original lengths, <u>the arc length shrinks (uncoil) according to the pressure</u>.



Elastic-type transducer: C-shaped Bourdon tube

The displacement of the moving end of the bourdon tube is measured to quantify the pressure

--mechanical linkage or gear system

--EM inductance

(LVDT: linear variable-differential transformer)

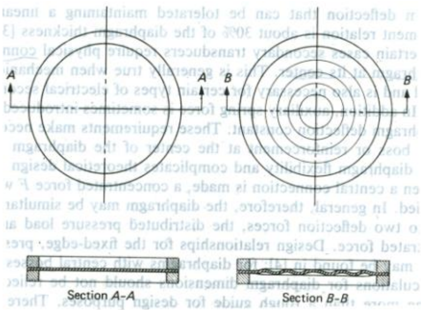
Good at static and quasi-static pressure measurements; However, <u>the mass of bourdon tube and LVDT limits the response frequency</u>. Thus, not suitable for use in highly-unsteady pressure fields.

Elastic-type transducer

[2] Diaphragm type:

Deformation element: a clamped circular plate (0—30,000 psi)

- Flat diaphragm for cases involving small deflection; often used in conjunction with electrical secondary transducers to increase the sensitivity.
- Corrugated type is particularly useful when larger deflections are encountered (increased range of linear deflection); two corrugated diaphragms are often joined at their edges to form “pressure capsule” →aneroid barometer (無液氣壓計)



- The volume of the sensing element should be minimized to provide reasonable dynamic response;
- High natural frequency is desired to capture unsteady pressure; linear deflection response is essential

Elastic-type transducer
—circular diaphragm of constant thickness

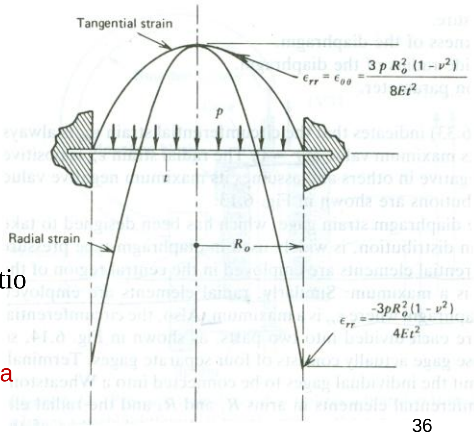
Strain distribution resulting from a uniform pressure on the face of a clamped circular plate of constant thickness:

$$\varepsilon_{rr} = \frac{3p(1-\nu^2)}{8E\delta^2}(R^2 - 3r^2)$$

$$\varepsilon_{\theta\theta} = \frac{3p(1-\nu^2)}{8E\delta^2}(R^2 - r^2)$$

R, δ : diaphragm radius, thickness

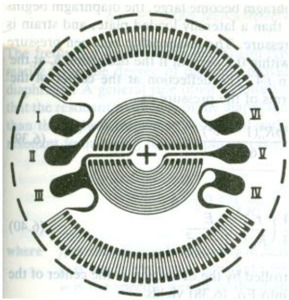
E, ν : Young's modulus and poisson's ratio



When the pressure range is even higher, a hollow cylinder is often employed as the sensing element (30,000 to 100,000 psi)

Elastic-type transducer
—circular diaphragm of constant thickness

Special purpose 4-element strain gage, taking advantage of the strain distribution: circumferential element are employed in the central region of the diaphragm where $\varepsilon_{\theta\theta}$ has the maximum; radial element is used near the edge where ε_{rr} is a minimum



$$\varepsilon_{rr} = \frac{3p(1-\nu^2)}{8E\delta^2}(R^2 - 3r^2)$$

$$\Rightarrow \varepsilon_{rr}\big|_{extreme} = \frac{3p(1-\nu^2)}{8E\delta^2}(-2R^2)\Big|_{\text{at r = R}}$$

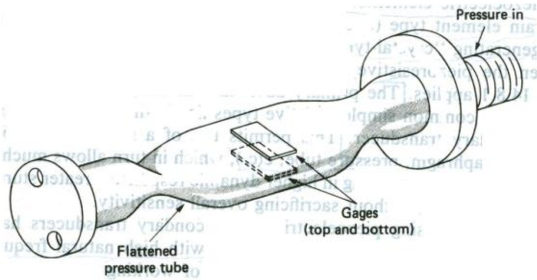
$$\varepsilon_{\theta\theta} = \frac{3p(1-\nu^2)}{8E\delta^2}(R^2 - r^2)$$

$$\Rightarrow \varepsilon_{\theta\theta}\big|_{extreme} = \frac{3p(1-\nu^2)}{8E\delta^2}(R^2)\Big|_{\text{at r = 0}}$$

Elastic-type transducer
—cylindrical pressure cell (high pressure)

Strain distribution resulting from a uniform pressure on the face of a clamped circular plate of constant thickness:

$$\frac{\Delta R}{P_i} = \frac{2FRd^2}{E}\left(\frac{2-\nu}{D^2-d^2}\right)$$



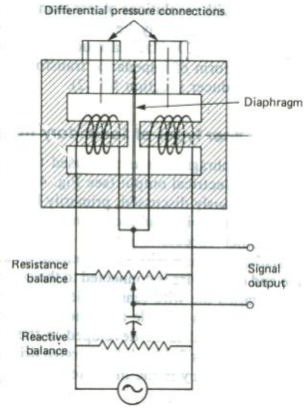
P_i : internal pressure;

ΔR: change in strain-gage resistance

E, ν : Young's modulus and poisson's ratio

d, D : inner / outer diameter of the cylinder

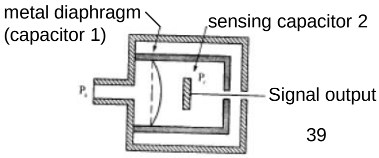
Inductive / Piezo-type pressure cell



- Variable inductance: deflection of metal diaphragm alters the relative inductance of the sensing block, which variation can be measured thru a simple circuit.
- Similarly, diaphragm deformation can be quantified by connection to piezo materials:

  (1) piezo-resistive element (passive, semiconductor strain element): increased sensitivity of the second transducer which allows us to use a less sensitive primary transducer (diaphragm)→stiffer construction and higher dynamic response (high natural frequency)

  (2) piezo-electric element (active, charge-generating crystal element)

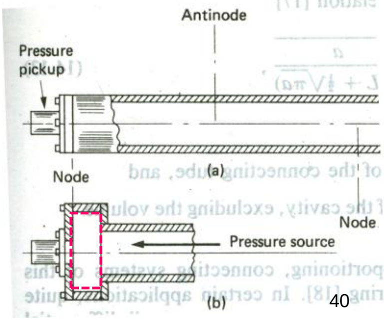


Dynamic characteristic of pressure-measuring system (gas-filled system)

- The response of a pressure-measuring system involves the signal pickup characteristics (electrical signals), the mechanical responses of the pipe system (connecting tube, passage ways), and the 'acoustic response' of the medium (acoustic resonance may occur).
- The dynamic response of sensing elements is often higher (since stiffer) than the (acoustic) resonance developed on the gas in the transmission passageways or connecting tubes. Thus, the influences on the medium dynamics may be neglected.
- The distance between adjacent nodes and antinodes = ¼ of the wavelength of the resonant frequency
- Medium response for gas-filled systems

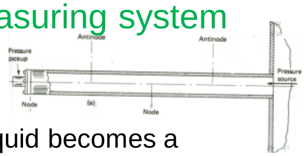
$$f = \frac{C}{4L}(2n-1); \qquad f = \frac{C}{2\pi}\sqrt{\frac{a}{V\left(L+\frac{1}{2\sqrt{\pi a}}\right)}}$$

C: sound speed in the 'pressured' medium
L: length of the tube; n: any positive integer
a: cross-sectional area of the connecting tube,
V: volume of the internal cavity (to install the sensor)



Dynamic characteristic of pressure-measuring system

(liquid-filled system)



- Since liquids are much denser than gases, the liquid becomes a major portion of the spring mass. Thus, the total liquid mass becomes a significant factor in determining the natural frequency of the system.

$$f = \frac{1}{2\pi}\sqrt{\frac{k^* g}{m^*}};\qquad \frac{1}{k^*} = \frac{1}{k_T} + \frac{1}{k_L}; m^* = m_T + m_L$$

k: mechanical stiffness ; m: mass
Subscript T / L: the moving element of the transducer, / the liquid

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$$m_L = \frac{4}{3}\rho_L a L\left(\frac{A}{a}\right)^2$$

a: cross-sectional area of the connecting tube,
L: length of the tube; n: any positive integer
A: effective area of the fluid under the influence of deflecting sensor element

$$A = \frac{\Delta V}{\Delta y}$$

ΔV: volume change accompanying the deflection of the sensing element
Δy: maximum displacement of the sensing element

Measurement of near vacuum environment

(1) McLeod gage 1874

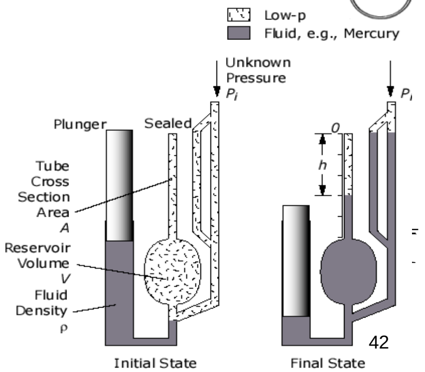
- Compressing the low pressure gas in a capillary tube (for accurate reading) by infilling mercury.
- Then compress the gas. By measuring the compression volume ratio, the initial pressure can be calculated through ideal gas law. (The final pressure can be measured from the mercury manometer.)

Boyle's law $p_1V_1 = p_2V_2$

$$p_1(V_0 + Ah_0) = p_2 Ah$$
$$= (\rho_{mercurry} gh + p_1)Ah$$

$$p_1(V_0 + A(h_0 - h)) = \rho_{mecurry} gAh^2$$

$$p_1 = \frac{\rho_{mecurry} gAh^2}{V_0 + A(h_0 - h)} \approx \frac{\rho_{mecurry} gAh^2}{V_0}$$



Near vacuum pressure measurement

Diluteness allows us to treat the gas molecules as individual particles—
apply **kinetic theory** for molecules momentum change.

(2) Langmuir Viscosity gauge

$$\frac{F}{A_{xz}} = \left(\frac{Pmv}{4kT}\right)\frac{U_1}{\beta} \quad \Rightarrow \quad p \propto v$$

$\beta \sim 1$ accounts for slip at surface

$$p\sqrt{m} = a\omega + b$$

Measuring plate oscillation frequency

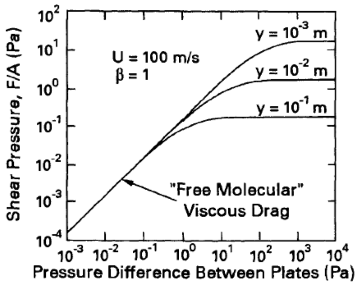


Fig. 2.7 Viscous shear force between two plates at 22°C.

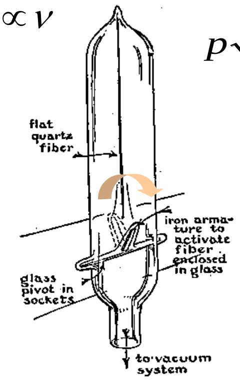
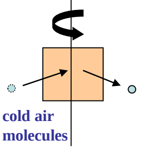
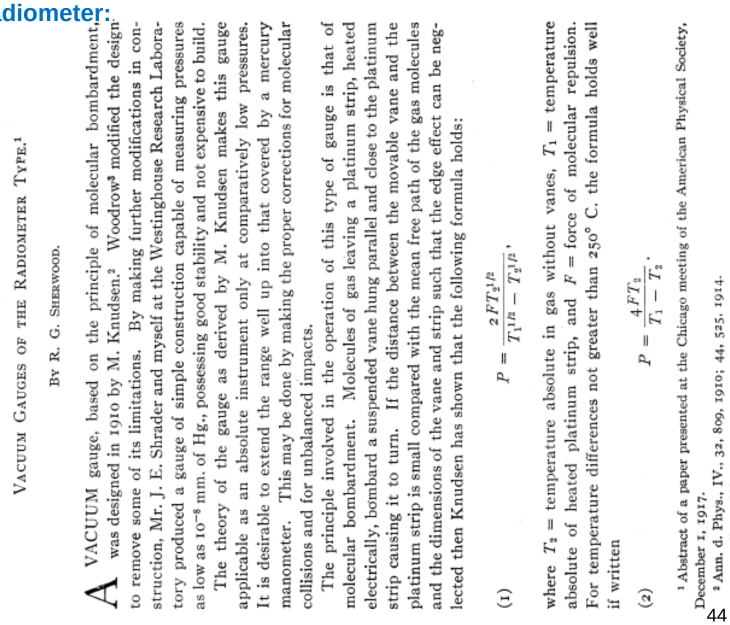


Fig. 40.

Near vacuum pressure measurement

(3) Knudsen radiometer:





VACUUM GAUGES OF THE RADIOMETER TYPE.[1]

BY R. G. SHERWOOD.

A VACUUM gauge, based on the principle of molecular bombardment, was designed in 1910 by M. Knudsen.[2] Woodrow[3] modified the design to remove some of its limitations. By making further modifications in construction, Mr. J. E. Shrader and myself at the Westinghouse Research Laboratory produced a gauge of simple construction capable of measuring pressures as low as 10^{-8} mm. of Hg., possessing good stability and not expensive to build.

The theory of the gauge as derived by M. Knudsen makes this gauge applicable as an absolute instrument only at comparatively low pressures. It is desirable to extend the range well up into that covered by a mercury manometer. This may be done by making the proper corrections for molecular collisions and for unbalanced impacts.

The principle involved in the operation of this type of gauge is that of molecular bombardment. Molecules of gas leaving a platinum strip, heated electrically, bombard a suspended vane hung parallel and close to the platinum strip causing it to turn. If the distance between the movable vane and the platinum strip is small compared with the mean free path of the gas molecules and the dimensions of the vane and strip such that the edge effect can be neglected then Knudsen has shown that the following formula holds:

$$P = \frac{2FT_2^{1/2}}{T_1^{1/2} - T_2^{1/2}}, \qquad (1)$$

where T_2 = temperature absolute in gas without vanes, T_1 = temperature absolute of heated platinum strip, and F = force of molecular repulsion. For temperature differences not greater than 250° C. the formula holds well if written

$$P = \frac{4FT_2}{T_1 - T_2}. \qquad (2)$$

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

[2] Ann. d. Phys., IV., 32, 809, 1910; 44, 525, 1914.

[3] Phys. Rev., IV., 6, 491, 1914.

If R_1 = electrical resistance of platinum strip when heated,
R_2 = electrical resistance at temperature of gauge,
K = constant of the gauge.
S = scale reading.

Then as a working formula (2) reduces to

$$P = \frac{KR_2}{R_1 - R_2}. \tag{3}$$

For the dimensions of the elements in the gauge designed for laboratory use, pressure above 10^{-4} mm. of the Hg cannot be measured with any degree of precision from the above formulæ without correcting for

(1) Collisions,
(2) Edge effect.

The following formula has been found to give the necessary corrections for one of these gauges up to 0.05 mm. of Hg on air.

$$P = \frac{\lambda}{d}\left(e^{\frac{d}{\lambda}} - 1\right) c \left(\frac{K \cdot S \cdot R_2}{R_1 - R_2}\right), \tag{4}$$

where λ = mean free path of the air molecules, d = distance between the movable vane and Pt heating strip. c = constant depending upon the ratio of the width of the suspended vane to d; the length of the vane being large as compared to the other dimensions is not considered in deriving the expression for the above correction, e = base of natural logarithms.

For greater pressures up to 1 or 2 cm. of Hg the gauge makes an ideal detector for small changes in pressure, but in its present form is not suitable for absolute measurements.

The sensibility of the gauge increases from nearly zero at about 2 cm. Hg to a maximum at about 0.05 mm. Hg; then decreases toward zero for very low pressures.

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